arXiv:gr-qc/9809081v4 25 Oct 1998

Energy-momentum tensor of quasiparticles in the effective gravity in superfluids.

G.E. Volovik
Low Temperature Laboratory, Helsinki University of Technology
P.O.Box 2200, FIN-02015 HUT, Finland
and
L.D. Landau Institute for Theoretical Physics, Moscow

February 7, 2008

Abstract

The problem of the energy-momentum conservation for matter in the gravitational field is discussed on the example of the effective gravity, which arises in superfluids. The "gravitational" field experienced by the relativistic-like massless quasiparticles which form the "matter" (phonons in superfluid 4He and low-energy fermions in superfluid 3He-A), is induced by the flow of the superfluid "vacuum". It appears that the energy-momentum conservation law for quasiparticles, has the covariant form $T^{\mu}_{\nu;\mu} = 0$. "Pseudotensor" of the energy-momentum for the "gravitational field" (superfluid condensate) appears to depend on "matter". In the presence of the stationary "gravitational" (superfluid) field the real thermodynamic temperature T is constant in the true dissipationless equilibrium state with no entropy production, while the "relativistic" temperature $T/\sqrt{g_{00}}$ is space dependent in agreement with Tolman's law. In the presence of the event horizon the true dissipationless equilibrium state does not exist. The quasiequilibrium dissipative motion across the horizon is considered. The inflationary stage of the expansion of the Universe can be modelled using the expanding Bose-condendsate.

Contents

1 *Introduction* **2**

2 *Hydrodynamics of superfluid liquid* **3**
- 2.1 *Basic equations.* 3
- 2.2 *Momentum conservation* 5

3 *Quasiparticles in effective gravity field* **5**
- 3.1 *"Relativistic" quasiparticles* 5
- 3.2 *Effective metric* 6
- 3.3 *Energy-momentum tensor* 7

4 *Thermal equilibrium* **8**
- 4.1 *True temperature and "relativistic" temperature* 8
- 4.2 *Global equilibrium in nonrelativistic liquid and in relativistic thermodynamics.* 8
- 4.3 *Energy-momentum tensor in quasiequilibrium.* 10
- 4.4 *The energy-momentum pseudotensor* 11

5 *Motion of normal component through horizon* **12**
- 5.1 *1-dimensional stationary "gravity" field* 12
- 5.2 *Absence of the global equilibrium state in the presence of horizon* 13
- 5.3 *Dissipative motion of normal component across horizon.* 14

6 *Simulation of Hubble expansion* **15**

7 *Conclusion.* **17**

1 *Introduction*

The effective gravity of the Sakharov type [1] can occur in many condensed matter systems. This allows us to use these systems for modelling the specific properties of the general relativity [2, 3, 4, 5, 6, 8, **?**]. The important class of the efective gravity theories, which naturally arises in condensed matter, is that which is induced by the gapless (massless) quasiparticles. An

example is the effective gravity in superfluid ^{3}He-A [8]. The energy spectrum of fermionic quasiparticles in this superfluid contains the topologically stable point nodes (Fermi points) [9]. In the vicinity of the gap node the quasiparticle spectrum becomes fully relativistic

$$g^{\mu\nu}(p_\mu - eA_\mu)(p_\nu - eA_\nu) = 0 \ . \tag{1}$$

The topological stability of the gap nodes is crucial: the small deformations of the quantum vacuum do not destroy the Fermi points but deform their characteristics, the slopes $g^{\mu\nu}$ and shifts A_μ in Eq.(1), which play the part of the gravitational field and different types of the gauge fields correspondingly. The gravity and gauge fields are thus the natural dynamical collective modes of the fermionic quantum vacuum with Fermi points.

Using such an effective gravity we discuss the particular problems related to the energy-momentum tensor of matter in the gravitational field.

2 *Hydrodynamics of superfluid liquid*

2.1 *Basic equations.*

For the energy-momentum problem the quasiparticle energy spectrum only is important, so that the quantum statistics of quasiparticles is irrelevant. So, we can consider the nomnrelativistic Landau two-fluid model of superfluidity, in which one component of the fluid is formed by free quasiparticles (Bose or Fermi) moving on the background of the superfluid vacuum play the part of matter. The effective gravitational field acting on quasiparticles is produced by the motion of the another component of the liquid – the superfluid condensate, which is characterized by the density ρ and superfluid velocity $\mathbf{v}_{(s)}$. The latter is not necessarily curl-free. Let us recall some basic equations of two-fluid dynamics, usung the simplest possible model with the following energy of the liquid [10]:

$$\mathcal{E} = \int d^3r \left(\frac{1}{2}\rho\mathbf{v}^2_{(s)} + \epsilon(\rho) + \sum_{\mathbf{p}} E(\mathbf{p},\mathbf{r})f(\mathbf{p},\mathbf{r}) \right) \ . \tag{2}$$

Here the first term is the kinetic energy of the superfluid condensate; the second term is its energy density as a function of ρ; $f(\mathbf{p},\mathbf{r})$ is the distribution

function of quasiparticles; $\sum_{\mathbf{p}} = \int d^3p/(2\pi\hbar)^3$ times the number of the spin degrees of freedom; the quasiparticle energy spectrum is:

$$E(\mathbf{p},\mathbf{r}) = E_{(0)}(\mathbf{p},\rho(\mathbf{r})) + \mathbf{p}\cdot\mathbf{v}_{(s)}(\mathbf{r}) \ . \tag{3}$$

where the second term is the Doppler shift in the moving condensate, and we assumed that the quasiparticle energy $E_{(0)}(\mathbf{p},\mathbf{r})$ in the superfluid frame (in the frame where the superfluid vacuum is at rest) depends on the coordinate only through the density ρ.

The mass current density or the linear momentum density consists of the vacuum current and the momentum of quasiparticles

$$\mathbf{j} = \rho\mathbf{v}_{(s)} + \mathbf{P} \ , \ \mathbf{P} = \sum_{\mathbf{p}} \mathbf{p} f(\mathbf{p},\mathbf{r}) \ , \tag{4}$$

so that the continuity equation is

$$\dot\rho + \vec\nabla(\rho\mathbf{v}_{(s)} + \mathbf{P}) = 0 \ . \tag{5}$$

The superfluid velocity of the vacuum obeys the London equation:

$$\dot{\mathbf{v}}_{(s)} + \vec\nabla\frac{\delta\mathcal{E}}{\delta\rho} - \frac{\mathbf{j}}{\rho}\times(\vec\nabla\times\mathbf{v}_{(s)}) = 0 \ . \tag{6}$$

In superfluid ^{4}He the superfluid velocity is potential, so that vorticity $\vec\nabla\times\mathbf{v}_{(s)}$ is nonzero only in the presence of quantized vortices. In superfluid ^{3}He-A the vorticity can be continuous and the London equation in this form can be obtained if one neglects the axial anomaly and the dependence of the energy on the anisotropy vector (see Eqs.(6.6)-(6.7) in Ref. [9]). The last term in equation (6) states that in the absence of anomalies the vorticity moves with the center-of-mass velocity $\mathbf{j}/\rho$.

The distribution function f of the quasiparticles is determined by the kinetic equation:

$$\dot f - \frac{\partial E}{\partial\mathbf{r}}\cdot\frac{\partial f}{\partial\mathbf{p}} + \frac{\partial E}{\partial\mathbf{p}}\cdot\frac{\partial f}{\partial\mathbf{r}} = J_{coll} \ . \tag{7}$$

The collision integral conserves the momentum and the energy of quasiparticles, i.e.

$$\sum_{\mathbf{p}}\mathbf{p}J_{coll} = \sum_{\mathbf{p}}E_{(0)}(\mathbf{p})J_{coll} = \sum_{\mathbf{p}}E(\mathbf{p})J_{coll} = 0 \ , \tag{8}$$

but not necessarily the particle number: as a rule the quasiparticle number is not conserved in superfluids.

2.2 *Momentum conservation*

From above equations one obtains the time evolution of the momentum density for each of two subsystems: the superfluid background (vacuum) and quasiparticles (matter). The momentum evolution of the superfluid vacuum is

$$\partial_t(\rho \mathbf{v}_{(s)}) = -\nabla_i(j_i \mathbf{v}_{(s)}) - \rho\vec{\nabla}\left(\frac{\partial \epsilon}{\partial \rho} + \sum_{\mathbf{p}} f \frac{\partial E_{(0)}}{\partial \rho}\right) + P_i \vec{\nabla} v_{(s)i} \ . \quad (9)$$

Using the kinetic equation Eq.(7) and condition $\sum_{\mathbf{p}} \mathbf{p} J_{coll} = 0$ from Eq.(8), one obtains the evolution of the momentum density of quasiparticles:

$$\partial_t \mathbf{P} = \sum_{\mathbf{p}} \mathbf{p}\partial_t f = -\nabla_i(v_{(s)i}\mathbf{P}) - \nabla_i\left(\sum_{\mathbf{p}} \mathbf{p} f \frac{\partial E_{(0)}}{\partial p_i}\right) - \sum_{\mathbf{p}} f \vec{\nabla} E_{(0)} - P_i \vec{\nabla} v_{(s)i} \ . \quad (10)$$

Though the momentum of each subsystem is not conserved because of the interaction with the other subsystem, the total momentum is conserved:

$$\partial_t j_i = \partial_t(\rho v_{(s)i} + P_i) = -\nabla_i P_{ik} \ , \quad (11)$$

with the stress tensor

$$P_{ik} = j_i v_{(s)k} + v_{(s)i} P_k + \sum_{\mathbf{p}} p_k f \frac{\partial E_{(0)}}{\partial p_i} + \delta_{ik} G \ , \ G = \rho\left(\frac{\partial \epsilon}{\partial \rho} + \sum_{\mathbf{p}} f \frac{\partial E_{(0)}}{\partial \rho}\right) - \epsilon \ . \quad (12)$$

3 *Quasiparticles in effective gravity field*

3.1 *"Relativistic" quasiparticles*

Let us consider the "relativistic" quasiparticles – phonons in superfluid ^{4}He or fermionic quasiparticles in superfluid ^{3}He. Their energy spectrum is linear in the limit of low energy:

$$E(\mathbf{p}, \mathbf{r}) = cp + \mathbf{p}\cdot\mathbf{v}_{(s)} \ , \ \text{or} \ (E - \mathbf{p}\cdot\mathbf{v}_{(s)})^2 - c^2 p^2 = 0 \ . \quad (13)$$

This equation is correct for phonons in superfluid ^{4}He, with c being the speed of sound. In the ^{3}He-A the low energy spectrum is also linear but

anisotropic (see [9]). The simplified equation (13) is valid in ^{3}He-A only for the homogeneous order parameter and is obtained after shifting of the momentum and rescaling. The function c in this case is some combination of the slopes of the energy spectrum.

3.2 *Effective metric*

Since the energy spectrum has the relativistic form, let us rewrite the time evolution of the quasiparticle momentum density in Eq.(10) in the covariant form, using the superfluid system as the effective background metric. The corresponding metric follows from the quasiparticle spectrum in Eq.(13), which can be written in a general Lorentzian form

$$g^{\mu\nu} p_\mu p_\nu = 0 \ , \tag{14}$$

where the metric elements are

$$g^{00} = 1 \ , \ g^{0i} = v_s^i \ , \ g^{ik} = -(c^2 \delta^{ik} - v_{(s)}^i v_{(s)}^k) \ . \tag{15}$$

and $p_0 = -E$, $p^0 = -E_{(0)} = -cp$, $p^\alpha p_\alpha = E_{(0)}^2 - c^2 p^2$.

The metric of the effective space in which quasiparticles propagate along the geodesics, i.e. $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = 0$, is correspondingly

$$g_{00} = \left(1 - \frac{v_{(s)}^2}{c^2}\right) \ , \ g_{0i} = \frac{v_{(s)i}}{c^2} \ , \ g_{ik} = -\frac{1}{c^2} \delta_{ik} \ . \tag{16}$$

Except for the conformal factor, it coincides with acoustic metric discussed for sound propagation in normal [2, 3] and superfluid [11] liquids. In our case the quasiparticles are not necessarily the phonons. The kinetic equation acquires the relativistic form

$$p^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial f}{\partial p^\alpha} = \tilde{J}_{coll} \ , \ \tilde{J}_{coll} = p^0 J_{coll} \ , \ p^0 = -E_{(0)} \ . \tag{17}$$

The system of quasiparticles play the part of the matter in general relativity. That is why one can expect that the momentum "conservation law" for matter has the usual covariant form

$$T^\mu_{\nu;\mu} = 0 \ , \quad \text{or} \quad \frac{1}{\sqrt{-g}} \partial_\mu \left(T^\mu_\nu \sqrt{-g} \right) - \frac{1}{2} T^{\alpha\beta} \partial_\nu g_{\alpha\beta} = 0 \ , \tag{18}$$

where $\sqrt{-g} = c^{-3}$. The terms which are not the total derivatives represent the forces acting on the matter from the effective gravitational field. Let us check if this equation can be applied to our case, where the effective gravity is induced by the superflow.

3.3 *Energy-momentum tensor*

Let us introduce the components of the energy-momentum tensor:

$$\sqrt{-g}T^0_i = -P_i \ , \ \sqrt{-g}T^0_0 = \sum_{\mathbf{p}} fE \ , \ \sqrt{-g}T^k_i = -\sum_{\mathbf{p}} p_i f \frac{\partial E}{\partial p_k} \ ,$$
$$\sqrt{-g}T^i_0 = P^i c^2 + v^i_{(s)} \sum_{\mathbf{p}} f E_{(0)} + v^k_{(s)} \sum_{\mathbf{p}} p_k f \frac{\partial E}{\partial p_i} \ . \qquad (19)$$

$$\sqrt{-g}T^{00} = \sum_{\mathbf{p}} f E_{(0)} \ , \ \sqrt{-g}T^{0i} = c^2 P^i + v^i_{(s)} \sum_{\mathbf{p}} f E_{(0)} \ ,$$
$$\sqrt{-g}T^{ik} = c^2 \sum_{\mathbf{p}} p_i f \frac{\partial E_{(0)}}{\partial p_k} + c^2 (v^k_{(s)} P^i + v^i_{(s)} P^k) + v^i_{(s)} v^k_{(s)} \sum_{\mathbf{p}} f E_{(0)} \ . \qquad (20)$$

Using the above expression for the quasiparticle energy-momentum tensor, one can be rewrite the equation for the quasiparticle momentum density, Eq.(10), and the analogous equation for the evolution of quasiparticles energy density in the following form:

$$\partial_\mu \left(\sqrt{-g} T^\mu_\nu \right) - \mathbf{P} \cdot \partial_\nu \mathbf{v}_{(s)} - \frac{\partial_\nu c}{c} \sum_{\mathbf{p}} f E_{(0)} = 0 \ . \qquad (21)$$

The last two terms are the forces acting on the subsystem of quasiparticles from the superfluid background, if the latter is inhomogeneous. These forces exactly reproduce the forces acting on the matter from the gravitational field in Eq.(18):

$$\frac{1}{2} \sqrt{-g} T^{\alpha\beta} \partial_\nu g_{\alpha\beta} = \mathbf{P} \cdot \partial_\nu \mathbf{v}_{(s)} + \frac{\partial_\nu c}{c} \sum_{\mathbf{p}} f E_{(0)} \ . \qquad (22)$$

Thus the energy-momentum tensor of quasiparticles satisfies the covariant equation (18). This result does not depend on the dynamic properties of the superfluid condensate (gravity field) and follows only from the "relativistic" spectrum of quasiparticles.

4 *Thermal equilibrium*

4.1 *True temperature and "relativistic" temperature*

In thermal equilibrium the quasiparticle distribution function is

$$f = \frac{1}{\exp \frac{E(\mathbf{p},\mathbf{r})-\mathbf{p}\cdot\mathbf{v}_{(n)}}{T} \pm 1} \ . \tag{23}$$

It is determined by the temperature T of the liquid and by the velocity $\mathbf{v}_{(n)}$ of the normal component of the liquid represented by the quasiparticles. This form of the distribution function is determined by the conservation of the energy and momentum by the collision term in kinetic equation. The Doppler shifted energy is

$$E(\mathbf{p},\mathbf{r}) - \mathbf{p}\cdot\mathbf{v}_{(n)} = cp - \mathbf{p}\cdot\mathbf{w} \ , \ \mathbf{w} = \mathbf{v}_{(n)} - \mathbf{v}_{(s)} \tag{24}$$

$\mathbf{w}$ is called the velocity of counterflow of the normal and superfluid fractions. Using these two velocities one can introduce the 4-velocities of the "matter", u^α and $u_\alpha = g_{\alpha\beta}u^\beta$, which satisfy equation $u_\alpha u^\alpha = 1$:

$$u^0 = \frac{1}{\sqrt{1-\frac{w^2}{c^2}}} \ , \ u^i = \frac{v^i_{(n)}}{\sqrt{1-\frac{w^2}{c^2}}} \ , \ u_i = -\frac{w_i}{c^2\sqrt{1-\frac{w^2}{c^2}}} \ , \ u_0 = \frac{1+\frac{\mathbf{w}\cdot\mathbf{v}_{(s)}}{c^2}}{\sqrt{1-\frac{w^2}{c^2}}} \ , \tag{25}$$

In terms of the 4-velocity the equilibrium distribution function has the relativistic covariant form

$$f = \frac{1}{\exp(-p_\mu u^\mu/T_{\rm eff}) \pm 1} \ , \ T_{\rm eff} = \frac{T}{\sqrt{1-\frac{w^2}{c^2}}} \ . \tag{26}$$

The temperature T is a real thermodynamic temperature, while the covariant "relativistic" temperature $T_{\rm eff}$ is not. This follows from the conditions for the global equilibrium, which means the absence of dissipation and entropy production.

4.2 *Global equilibrium in nonrelativistic liquid and in relativistic thermodynamics.*

The global equilibrium state in nonrelativistic superfluids is determined by the following conditions for the absence of dissipation [10, 15]:

$$T = const \ , \tag{27}$$

$$\mathbf{v}_{(n)} = \mathbf{a} + \mathbf{b} \times \mathbf{r} \ , \ \mathbf{a} = const \ , \ \mathbf{b} = const \ , \tag{28}$$

$$\vec{\nabla} \cdot (\mathbf{j} - \rho \mathbf{v}_{(n)}) = 0 \ , \tag{29}$$

$$\frac{\delta \mathcal{E}}{\delta \rho} + \mathbf{v}_{(s)} \cdot \mathbf{v}_{(n)} = const \ . \tag{30}$$

In the presence of the stationary inhomogeneous "gravitational field" (i.e. in the presence of the fields $\mathbf{v}_{(s)} = \mathbf{v}_{(s)}(\mathbf{r})$, $c = c(\mathbf{r})$ and $\rho = \rho(\mathbf{r})$) the last 3 conditions require $\mathbf{v}_{(n)} = 0$ in the frame where the superfluid (gravity) fields do not depend on time. Let us consider this on example of Eq.(29). According to continuity equation Eq.(5), in the stationary case one has $\vec{\nabla} \cdot \mathbf{j} = 0$. Then using the Eq.(28), one can see that the condition in Eq.(29) becomes $\mathbf{v}_{(n)} \cdot \vec{\nabla} \rho = 0$. Thus if ρ depends on $\mathbf{r}$, the requirement for equilibrium is $\mathbf{v}_{(n)} = 0$. Then if $\mathbf{v}_{(s)}$ depends on $\mathbf{r}$, the condition $\mathbf{v}_{(n)} = 0$ follows from Eq.(30). Thus in the "gravitational field" ($\mathbf{v}_{(s)} = \mathbf{v}_{(s)}(\mathbf{r})$, $\rho = \rho(\mathbf{r})$) one always has $\mathbf{v}_{(n)} = 0$ as the global equilibrium condition. In general it means that if any order parameter texture moves with respect to the normal component, there is a dissipative friction between the texture and quasiparticle system.

Thus the true equilibrium (nondissipative) state in the presence of the stationary "gravitational field" is characterized by the constant real temperature T, while the covariant "relativistic" temperature changes in space as $T_{\rm eff}(\mathbf{r}) = T/\sqrt{1 - \mathbf{v}^2_{(s)}(\mathbf{r})/c^2(\mathbf{r})} = T/\sqrt{g_{00}(\mathbf{r})}$ in agreement with Tolman's law [13]. This will be illustrated further below for the one-dimensional texture.

Without the "gravity" field, i.e. at constant ρ, c and $\mathbf{v}_{(s)}$, the 6 equilibrium conditions above can be expressed in the relativistic form using the relativistic temperature vector $\beta_\mu = u_\mu / T_{\rm eff}$:

$$\partial_\nu \beta_\mu + \partial_\mu \beta_\nu = 0 \ , \ \beta_\mu = \frac{u_\mu}{T_{\rm eff}} \ . \tag{31}$$

The covariant extension of the global equilibrium condition in Eq.(31) to the case of the stationary "gravity" field (i.e. ρ, c and $\mathbf{v}_{(s)}$ are functions of $\mathbf{r}$) is that the temperature vector β_μ is the timelike Killing-vector (see Ref.[16] and references there):

$$\beta_{\mu;\nu} + \beta_{\nu;\mu} = 0 \ . \tag{32}$$

It is easy to check for the 1-dimensional case discussed later, that the condition $\mathbf{v}_{(n)} = 0$ and $T = const$ for the global equilibrium of the liquid is consistent with the covariant global equilibrium equation (32), unless an

event horizon is present. The thermodynamic equilibrium in the presence of the black hole horizon has been discussed in detail in [17].

4.3 *Energy-momentum tensor in quasiequilibrium.*

In quasiequilibrium states T and $\mathbf{v}_{(n)}$ are the local hydrodynamic quantities which depend on space and time coordinates. The quasiparticle momentum density in a local equilibrium is

$$\mathbf{P}(\mathbf{r},t)=\sum_{\mathbf{p}}\mathbf{p}f(\mathbf{p},\mathbf{r},t)=\rho_n^{(0)}\frac{\mathbf{w}}{\left(1-\frac{w^2}{c^2}\right)^3}~, \tag{33}$$

where $\rho_n^{(0)}$ is normal fluid density in the limit of zero counterflow, $\mathbf{w}=0$. For the spin-1/2 fermions

$$\rho_n^{(0)}=\frac{7}{90}\pi^2\frac{T^4}{c^5}=\frac{7}{90}\pi^2\frac{T^4}{c^2}\sqrt{-g}~. \tag{34}$$

The quasiparticle energy density is

$$\sum_{\mathbf{p}}E_{(0)}f=\frac{\varepsilon^{(q)}+P^{(q)}}{1-\frac{w^2}{c^2}}-P^{(q)}=\frac{3}{4}\rho_n^{(0)}c^2\frac{1+\frac{1}{3}\frac{w^2}{c^2}}{\left(1-\frac{w^2}{c^2}\right)^3}~, \tag{35}$$

where $P^{(q)}$ is the quasiparticle contribution to pressure

$$P^{(q)}=\frac{1}{4}\frac{\rho_n^{(0)}c^2}{\left(1-\frac{w^2}{c^2}\right)^2}~,~\frac{\partial P^{(q)}}{\partial\mathbf{v}_n}=\frac{\partial P^{(q)}}{\partial\mathbf{w}}=\mathbf{P}~. \tag{36}$$

The quasiparticle contribution to the stress tensor is

$$\sum_{\mathbf{p}}p_kf\frac{\partial E_{(0)}}{\partial p_i}=(\varepsilon^{(q)}+P^{(q)})\frac{w_iw_k}{c^2-w^2}+P^{(q)}\delta^i_k~. \tag{37}$$

where $\varepsilon^{(q)}=3P^{(q)}=\rho^{(q)}c^2$ is the energy density of "matter" and $\rho^{(q)}$ is its "mass".

Combining the above equations, one can write the energy-momentum tensor for quasiparticles in the general "relativistic" form using the 4-velocity introduced in Eq.(25):

$$T^{\mu\nu}=(\varepsilon+P)u^\mu u^\nu-Pg^{\mu\nu}~. \tag{38}$$

where ε is the "covarant" energy density of quasiparticles: $\varepsilon = \varepsilon^{(q)}/\sqrt{-g}$. For the 1/2-spin fermionic quasipaticles one has

$$\varepsilon = \frac{\varepsilon^{(q)}}{\sqrt{-g}} = \frac{7}{120}\pi^2 T_{\rm eff}^4 = \frac{7}{120}\pi^2 \frac{T^4}{\left(1-\frac{w^2}{c^2}\right)^2}~,~P=\frac{\varepsilon}{3}~. \tag{39}$$

The relevant components of the energy-momentum tensor are

$$T_i^0 = -(\varepsilon+P)\frac{w_i}{c^2-w^2}~,~T_i^k = -(\varepsilon+P)\frac{v^k_{(n)}w_i}{c^2-w^2} - P\delta_i^k~,$$

$$T_0^i = (\varepsilon+P)v^i_{(n)}\frac{c^2+\mathbf{w}\cdot\mathbf{v}_{(s)}}{c^2-w^2}~,~T^{00} = \frac{(\varepsilon+P)}{1-w^2/c^2} - P~,~T_0^0 = T^{00} + \frac{\mathbf{P}\cdot\mathbf{v}_{(s)}}{\sqrt{-g}}~. \tag{40}$$

4.4 *The energy-momentum pseudotensor*

The equation for the energy-momentum of quasiparticles is covariant but this equation does not represent any conservation law. On the contrary, the total energy and momentum of the system, quasiparticles + superfluid condensate (gravity field), are conserved, but this conservation laws cannot be expressed in the covariant form. This is another problem of the general relativity, which probably also can be attacked using the condensed matter analogy. In the condensed matter the energy-momentum tensor of the system exists on the microscopic level, i.e. it can be expressed using the microscopic variables. But in many cases, for example in ferromagnets [12], it cannot be expressed in terms of the local effective variables, such as magnetization. The effective gravity field as the local low-energy collective mode can be just an another example.

Let us consider the analog of the pseudotensor of the energy-momentum of the gravitational field, which is in our case the part of the total energy momentum tensor coming from the superfluid motion of the condensate. The conservation law for the total energy-momentum can be written as

$$\partial_\mu\left(\sqrt{-g}T^\mu_\nu + \sqrt{-g}t^\mu_\nu\right) = 0~, \tag{41}$$

where t^μ_ν is the pseudotensor of energy-momentum. From the basic equations for superfluids it follows that

$$\sqrt{-g}t^0_i = -\rho v_{(s)i}~,$$

$$
\sqrt{-g}t_i^k = -j^k v_{(s)i} - \delta_i^k \left[\rho \left(\frac{\partial \epsilon}{\partial \rho} + \sum_{\mathbf{p}} f \frac{\partial E_{(0)}}{\partial \rho} \right) - \epsilon \right] ,
$$

$$
\sqrt{-g}t_0^0 = \frac{1}{2}\rho \mathbf{v}_{(s)}^2 + \epsilon(\rho) ,
$$

$$
\sqrt{-g}t_0^i = j^i \left(\frac{\partial \epsilon}{\partial \rho} + \sum_{\mathbf{p}} f \frac{\partial E_{(0)}}{\partial \rho} + \frac{\mathbf{v}_{(s)}^2}{2} \right) . \tag{42}
$$

It appears, that the pseudotensor of the "gravitational field" depends on matter.

Let us introduce the Lagrangian

$$
L = \frac{1}{2}\rho \mathbf{v}_{(s)}^2 + \epsilon(\rho) + \sum_{\mathbf{p}} Ef + \rho \partial_t \Phi . \tag{43}
$$

where Φ is the phase of the condensate related to the superfluid velocity:

$$
\vec{\nabla}\Phi = \frac{m}{\hbar}\mathbf{v}_{(s)} . \tag{44}
$$

Here m is the mass of the initial boson in the superfluid Bose condensate, and further we put $\hbar/m = 1$. Then the above pseudotensor of the "gravitational field" can be written in the compact form:

$$
\sqrt{-g}t_\nu^\mu = L\delta_\nu^\mu - \frac{\partial L}{\partial \nabla_\mu \Phi}\nabla_\nu \Phi - \delta_\nu^\mu \sum_{\mathbf{p}} Ef . \tag{45}
$$

5 *Motion of normal component through horizon*

5.1 *1-dimensional stationary "gravity" field*

Let the superfluid-gravity field is stationary and depends only on one space coordinate x. The motion of the "matter" is determined by Eq.(21), which using Eq.(40) and $\mathbf{w} = \hat{\mathbf{x}}w(x)$, $\mathbf{v}_{(s)} = \hat{\mathbf{x}}v(x)$ read:

$$
\sqrt{-g}T_0^x = const = 4P^{(q)} \frac{(v+w)(c^2+wv)}{c^2-w^2} , \tag{46}
$$

$$
-\partial_x \left(4P^{(q)} \frac{(v+w)w}{c^2-w^2} + P^{(q)} \right) = \frac{4P^{(q)}w}{c^2-w^2}\partial_x v + \frac{\partial_x c}{c} \left(4P^{(q)} \frac{c^2}{c^2-w^2} - P^{(q)} \right) . \tag{47}
$$

The Eq.(46) manifests the constant energy flux along x axis. The flux is zero if the normal component velocity is at rest in the frame of the texture, i.e. if $\mathbf{v}_{(n)} = 0$, or $w(x) = -v(x)$. In this case the second equation Eq.(47) is satisfied if the temperature T is constant, so that

$$P^{(q)}(x) \propto \frac{T^4}{c(x)^3}\left(1 - \frac{v^2(x)}{c^2(x)}\right)^{-2} . \tag{48}$$

This solution describes a true equilibrium state in the presence of the "gravity field": there is no dissipation (the conditions for the absence of dissipation is $\mathbf{v}_{(n)} = 0$ and $T = const$). Note that the relativistic temperature T_{eff} depends on x in this equilibrium state. The covariant equilibrium condition $\beta_{\mu;\nu} = 0$ is satisfied.

5.2 *Absence of the global equilibrium state in the presence of horizon*

Let us consider the motion of quasiparticle system in the presence of the event horizon. For simplicity we assume the constant velocity $\mathbf{v}_{(s)} = \hat{\mathbf{x}}v$ and the coordinate dependent speed of sound $c(x)$, so that at some point $x = x_0$ one has $c^2(x_0) = v^2$. Since $g_{00} = 1 - v^2_{(s)}/c^2$ in Eq.(16) changes sign at $x = x_0$ one has a horizon at this point. In the simplest realization of the horizon the speed of "light" changes across the horizon in the following way:

$$c(x) = v(1 - a \tanh(bx)) . \tag{49}$$

Here a and b are constant parameters and the position of horizon is chosen at $x = 0$.

Outside the horizon one can determine the true equilibrium state with $\mathbf{v}_{(n)} = 0$ (or $w = -v$) and $T = const$. However, this state cannot be determined globally is the whole space: the "relativistic" temperature

$$T_{\text{eff}} = \frac{T}{\sqrt{g_{00}}} \ , \ g_{00} = 1 - \frac{v^2}{c^2} \ , \tag{50}$$

which determines the "relativistic" pressure $P^{(q)}$, diverges at the horizon together with the energy density and cannot be extended across the horizon.

Thus in the presence of a horizon, one must look for the quasiequilibrium solution with the space dependent $T(x)$ and $w(x)$ and thus with dissipation.

In general relativity the vacuum energy (the energy of the Boulware vacuum) also diverges at horizon, but with the negative sign. In the Hartle-Hawking state, in which the thermodynamic temperature T equals the Hawking temperature $T_{\rm H} = (\hbar/2\pi)(dc/dx)|_{\rm hor}$ the two divering terms in the stress-energy tensor cancel each other (see references in the recent paper [18]). However in condensed matter such a diveregence of the energy density in each of the two subsystems (quasiparticles, which form the "matter", and superfluid condensate, which produces the effective gravity field) represents the real physical singularity at horizon. The external observer who lives in the Galilean world of the physical laboratory can use the "superluminal" signals to measure the quasiparticle distribution function both outside and inside the horizon. He will find that the quasiparticle energy density does really diverge at horizon, so that the global equilibrium does not exist. Such a singularity can be avoided either by escaping from the global equilibrium to the local one with dissipation, or by consideration of the high-energy nonrelativistic corrections to the quasiparticle spectrum, together with the reconstruction of the superfluid vacuum within the horizon, which again leads to dissipation.

5.3 *Dissipative motion of normal component across horizon.*

In the presence of horizon, the energy flux in Eq.(46) is no more zero and one can express $P^{(q)}$ from this equation

$$P^{(q)} \propto \frac{c^2 - w^2}{(v+w)(c^2+vw)} \ . \tag{51}$$

and insert it to Eq.(47) to obtain the following equation for w (note that we assumed $v = const$ for the superfluid velocity):

$$-\partial_x \left(\frac{4vw + 3w^2 + c^2}{(v+w)(c^2+vw)} \right) = \frac{\partial_x c}{c} \left(\frac{w^2 + 3c^2}{(v+w)(c^2+vw)} \right) \ . \tag{52}$$

It follows that w depend on x as a function of $c(x)$, i.e. $w = w(c(x))$, where $w(c)$ satisfies the equation

$$-c\frac{d}{dc}\left(\frac{3w}{c^2+vw} + \frac{1}{v+w} \right) = \frac{w^2+3c^2}{(v+w)(c^2+vw)} \ . \tag{53}$$

Introducing $c = vC$ and $w = WvC$, one obtains

$$-\frac{d}{dC}\left(\frac{3W}{C+W}+\frac{1}{1+CW}\right)=\frac{W^2+3}{(1+CW)(C+W)}\ . \tag{54}$$

This equation has a particular solution $W = -1$, or $w = -c$, which according to Eq.(51) corresponds to zero temperature, $T = 0$.

The function $C(x)$, which gives the horizon at $x = 0$ in Eq.(49), is

$$C = 1 - a\tanh(bx)\ . \tag{55}$$

If $a \ll 1$ and thus $|C-1| \ll 1$, one has the following solution of this equation

$$W = -\frac{3}{2}(C-1)\ . \tag{56}$$

This solution has no singularity at the horizon ($x = 0$). The temperature T and the counterflow velocity w slightly change across the horizon. Since the normal velocity $v_{(n)} = w + v_{(s)} \approx v_{(s)}$ is nonzero in the horizon frame, this state in Eq.(56) is in local thermal equilibrium, but not in a global one. Therefore there is a dissipation.

Let us consider another extreme case $|W+1| \ll 1$, when the velocity of the normal component is small in the horizon frame. This is close to the Boulware vacuum in the general relativity. In this regime the solution of Eq.(54) is

$$W = -1 + A(C-1)^2\ , \tag{57}$$

where A is an arbitrary parameter and it is assumed that $|C-1| \ll 1$. Since $c^2 - w^2 \propto A(c-v)^2$, $v + w \propto -(c-v)$ and $c^2 + wv \propto (c-v)$, one has $T^4 \propto (c^2-w^2)^3/(v+w)(c^2+wv) \propto (c-v)^4$, i.e. $T \propto |c-v|$ and $T_{\rm eff} = T/\sqrt{1-w^2/c^2} = const.$ In this solution the real temperature T is space dependent and is zero at horizon, while the effective relativistic temperature $T_{\rm eff}$ is constant across the horizon.

6 *Simulation of Hubble expansion*

The expanding solution of the motion equations is

$$\mathbf{v}_{(s)}(\mathbf{r},t) = \mathbf{v}_{(n)}(\mathbf{r},t) = \tilde{H}(t)\mathbf{r}\ ,\ \rho = \rho(t)\ . \tag{58}$$

The Hubble parameter and its connection to the density of the liquid are found from Eqs.(5,6):

$$\tilde{H}(t) = -\frac{\partial_t \rho}{3\rho} \ , \ \partial_t \tilde{H} + \tilde{H}^2 = 0 \ . \tag{59}$$

which give

$$\tilde{H}(t) = (t - t_0)^{-1} \ , \ \rho(t) = \mathrm{Const}\ (t - t_0)^{-3} \tag{60}$$

Note that the Hubble parameter in the radiation-dominated Universe is $H(t) = (1/2)(t - t_0)^{-1} = (1/2)\tilde{H}(t)$.

Since the quasiparticle momentum $\mathbf{P} = 0$ (because $\mathbf{v}_{(s)} = \mathbf{v}_{(n)}$), the Eq.(10) is automatically satisfied, while the energy conservation law for quasiparticles gives the following equation for the quasiparticles energy density $\varepsilon^{(q)} = 3P^{(q)}(= \rho^{(q)} c^2)$:

$$\partial_t \varepsilon^{(q)} + \left(4\tilde{H} - \frac{\partial_t c}{c}\right) \varepsilon^{(q)} = 0 \ . \tag{61}$$

Since $\varepsilon^{(q)} = \varepsilon/c^3 \propto T^4/c^3$, one obtains equation for T:

$$\partial_t \left(\frac{T}{c}\right) + \tilde{H} \left(\frac{T}{c}\right) = 0 \ . \tag{62}$$

which gives $T(t) \propto c(t)/(t - t_0)$.

The time dependence of the speed of light is determined by its dependence on the density ρ: $c(t) = c(\rho(t))$. The cosmological scenario of the expansion of the radiation-dominated Universe, with $T(t) \propto (t - t_0)^{-1/2}$, is obtained if $c \propto (t - t_0)^{1/2}$. If the speed c comes from the compressibility of the superfluid vacuum, then such behavior of c can result from the following equation of states for the superfluid condensate: $\epsilon(\rho) \sim \rho^{2/3}$ which gives $c(\rho) \sim \rho^{-1/6} \propto (t - t_0)^{1/2}$. Note that in the cases of the Bose condensation of the almost ideal Bose gas and of the superfluidity of the almost ideal Fermi gas, the equation of states are corespondingly $\epsilon(\rho) \sim \rho^2$ and $\epsilon(\rho) \sim \rho^{5/3}$ with $c(\rho) \sim \rho^{1/2} \propto (t - t_0)^{-3/2}$ and $c(\rho) \sim \rho^{1/3} \propto (t - t_0)^{-1}$.

If one introduces the comoving coordinate frame: $\mathbf{r} = \tilde{\mathbf{r}}\tilde{a}(t)$ with $\partial_t \tilde{a} = \tilde{a}\tilde{H}$, one obtains the following effective metric in this frame

$$ds^2 = dt^2 - a^2 d\tilde{\mathbf{r}}^2 \ , \ a = \frac{\tilde{a}}{c} \ , \ \frac{\partial_t \tilde{a}}{\tilde{a}} = \tilde{H} \ . \tag{63}$$

Since $\tilde{a} \propto t - t_0$, the behavior corresponding to the radiation-dominated Universe occurs if $c \propto (t-t_0)^{1/2}$. Then $a = \tilde{a}/c \propto (t-t_0)^{1/2}$ and $\partial_t a/a = H = (1/2)\tilde{H}$. Such case occurs if $\epsilon(\rho) \sim \rho^{2/3}$.

Another special case is when $c \propto t - t_0$. In this case $a = \mathrm{Const}$, and $T = \mathrm{Const}$. This means that the effective space-time for quasiparticles is Minkowski flat space-time, inspite of the nontrivial dynamics of the background superfluid condensate. Such case occurs if $\epsilon(\rho) \sim \rho^{1/3}$ which gives $c(\rho) \sim \rho^{-1/3} \propto t - t_0$.

In the case of the superfluidity of the almost ideal Bose and Fermi gases, the integral $\int dt\ a^{-1}(t)$ diverges at the origin. Thus both cases correspond to the power-law inflation.

7 *Conclusion.*

The next step is to derive the effective action for the "gravity field". The effective gravity theory is obtained by integration over the fermionic degrees of freedom in the presence of the background field [1]. The latter field becomes dynamical and represents the low-frequency collective modes of the system of interacting particles – ^{3}He or ^{4}He atoms. In the case discussed here the result is well known: the hydrodynamic equations for the background superfluid vacuum in Eqs.(5,6), which do not depend on the details of the microscopic interactions between the atoms, but which are far from being covariant and relativistic. To have the relativistic equations for the effective gravity field one must consider such hypothetical condensed mater systems where the main contribution to the path integral comes from the "relativistic" quasiparticles. If in addition these fermions or bosons are "massless", there is an extra symmetry: with respect to the multiplication of the metric by an arbitrary function $g_{\mu\nu} \rightarrow a^2 g_{\mu\nu}$. Such conformal symmetry of quasiparticles should lead to the conformal invariant and covariant action for the superfluid condensate, which contains the Weyl tensor (see e.g. [14]).

In a real condensed matter system the covariant terms in the superfluid action mainly represent the corrections to the leading hydrodynamic terms (see e.g. [11]). However for some specific phenomena, for which the contribution of the low-energy "relativistic" tail is dominating, the covariant and even conformal terms can be important: anomaly is one of the examples. In condensed matter the energy cut-off parameter, at which the low-energy

"relativistic" behavior transforms to the high-energy Galilean one, is the physical parameter and reflects the fact that the curved space in the low-energy edge is the effective space, while the underlying coordinate space is pure Galilean. In this flat space the thermodynamic temperature T is well defined. It is constant in the global equilibrium and is viewed by the "relativistic" low-energy quasiparticles as the constant in the Tolman' law for the local "relativistic" temperature: $T_{\rm eff}(\mathbf{r})\sqrt{g_{00}(\mathbf{r})} = const = T$. The fact that it is the temperature T, which is of the physical significance, rather than $T_{\rm eff}$, makes physical the divergence of the quasparticle energy density at horizon. As a result, in the presence of the event horizon the global thermodynamic equilibrium of the condensed matter system is impossible: one always has the entropy production when the normal component of the liquid moves across the horizon.

I am grateful to Dmitrij Fursaev, Ted Jacobson, Alexei Morozov and Alexei Starobinsky for discussions.

References

[1] A.D. Sakharov, Sov. Phys. Doklady **12**, 1040 (1968).

[2] W.G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981).

[3] M. Visser, Class. Quant. Grav. **15**, 1767 (1998).

[4] M.O. Katanaev and I.V. Volovich, "Scattering on Dislocations and Cosmic Strings in the Geometric Theory of Defects", gr-qc/9801081

[5] R. Bausch and R. Schmitz, Phys. Rev. Lett., **80**, 2257 (1998).

[6] T. Jacobson and G.E. Volovik, Phys. Rev. **D 58**, 064021 (1998).

[7] T. Jacobson and G.E. Volovik, "Effective spacetime and Hawking radiation from moving domain wall in thin film of ^{3}He-A", to be published.

[8] G.E. Volovik, "Induced Gravity in Superfluid ^{3}He", cond-mat/9806010.

[9] G. E. Volovik, *"Exotic properties of superfluid ^{3}He"*, World Scientific, Singapore - New Jersey - London - Hong Kong, 1992.

[10] I.M. Khalatnikov, "An Introduction to the Theory of Superfluidity", (Bemjamin, New York), 1965.

[11] K.N.Ilinski and A.S.Stepanenko, "From Bose condensation to quantum gravity and back", cond-mat/9803233.

[12] G.E. Volovik, J. Phys. C **20**, L83 (1987).

[13] R.C. Tolman, *Relativity, Thermodynamics and Cosmology* (Clarendon, Oxford, 1934).

[14] N.D. Birell and P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, 1982).

[15] G.E. Volovik, Soviet Phys. Usp. **27**, 363 (1984).

[16] W. Zimdahl, "Cosmological particle production and generalized thermodynamic equilibrium", gr-qc/9711081.

[17] S.A. Hayward "Unified first law of black-hole dynamics and relativistic thermodynamics", gr-qc/9710089.

[18] S. Mukohyama and W.Israel, Phys. Rev. **D 58**, 104005 (1998).